

Polaritonics in nanowires made from dispersive materials

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(Received 19 May 2009; revised manuscript received 15 July 2009; published 3 November 2009)

We have studied the optoelectronics of a polaritonic nanowire which is fabricated by embedding a polaritonic crystal into another polaritonic crystal. It is considered that the band gap of the embedded crystal lies within the band gap of the host crystal. This band-gap engineering is satisfied by GaP and MgO crystals where MgO is the host crystal. Polaritons in the nanowire are confined within the embedded crystal. Bound states of the confined polaritons are calculated using the transfer-matrix method. The bound polariton energies are evaluated for a GaP-MgO nanowire. It is found that the number of bound states in the wire depends on its size, well depth, and the barrier height. The absorption coefficient of the system has also been calculated by using the time-dependent Schrödinger equation method. Numerical simulations for the GaP-MgO nanowire show that when the resonant energy of a quantum dot lies near the bound states the spectrum has several transparent states. The nanowire can be switched among the transparent or absorbing states by tuning the resonant state of the quantum dot. The present findings can be used to make types of polaritonic devices such as polaritonic switches and transistors.

DOI: [10.1103/PhysRevB.80.195303](https://doi.org/10.1103/PhysRevB.80.195303)

PACS number(s): 73.21.-b

I. INTRODUCTION

Recently there has been considerable interest in studying the properties of polaritonic materials.¹⁻¹¹ Examples of these materials include semiconductors (i.e., GaP, GaAs, and SiC), oxide crystals (i.e., MgO), and salts. They are also called dispersive materials.¹⁻³ Polaritonic materials have energy gaps in their dispersion relation due to the coupling between optical phonons and photons.¹⁻⁴ In this paper we study the optoelectronic properties of nanowires made from polaritonic materials.

In polaritonic materials the radiation signals are carried out by an admixture of photons and optical phonons rather than electrons or photons. An admixture of photons and optical phonons create a new quantum particle called polaritons. The study of characterizations and applications of these materials is called “polaritonics.” Polaritons propagate with frequencies in the range of hundreds of gigahertz to several terahertz (THz). Therefore, polaritonics lies in an intermediate regime between photonics and electronics.

The new field of polaritonics is going to be useful because electronics suffers technological and physical barriers to increase speed whereas photonics requires lossy integration of a light source and guiding structures.^{5,6} Therefore, it bridges the gap between electronics and photonics. It has a wide range of applications, including high bandwidth signal processing, THz imaging, and THz spectroscopy.^{5,6}

Rupasov and Singh¹ have shown that there is the suppression of spontaneous emission when a quantum dot is placed within polaritonic materials. We found a polariton-dot bound state in which the polaritons are localized in the vicinity of the dot. We have also shown that bound polariton (BP) solitons can be created in these materials.¹ Lau and Singh² have studied the spontaneous emission rate of polaritons in III-V semiconductors when two two-level atoms are doped. It is found that when the two atoms are very close to each other the degenerate states split into a symmetric and an antisymmetric states. The system in the symmetric state can radiate a

polariton with a probability that is twice that of the independent dot case. This is known as super-radiance.

Nonlinear two-polariton absorption has also been studied in polaritonic materials doped with an ensemble of three-level quantum dots.³ It was considered that the quantum dots are interacting with each other via the dipole-dipole interaction. It has been found that two-photon absorption can be turned on and off when the decay resonance energy of the three-level quantum dots is moved within the lower energy band. The inhibition of two-polariton absorption can also be achieved by controlling the strength of the dipole-dipole interaction.

Polaritonic waveguides and resonators are also fabricated through the femtosecond laser machining of holes or trenches which are carved through LiNbO₃ or LiTaO₃ host crystals.⁵⁻⁷ Infrared and optical 111 polaritons propagation have also been investigated in a polaritonic wire made from an isotropic dielectric material coated with metal.⁸ The wire contains surface bound modes which did not exist previously in the dielectric material.

Polaritons have also been studied in periodic and quasiperiodic multilayers made up of both positive (SiO₂) and negative refractive index materials.⁹ Polaritonic band gaps have been found in these structures. The reflection and absorption measurements for polaritons have been performed in ferroelectric crystals in the terahertz region.¹⁰ Photonic crystals have also been fabricated by using polaritonic materials.¹¹

In the present paper polaritonic nanowires are fabricated by embedding a polaritonic material into another. A schematic diagram for the nanowire is shown in Fig. 1. It is considered that the embedded polaritonic material has a smaller band gap than the host material. For example, the semiconductor GaP has a smaller band gap than that of MgO (see Fig. 2). Therefore, a nanowire can be fabricated by embedding GaP into MgO. Because of this band-gap engineering, the polaritons are confined in the embedded materials and have bound states. The radius of the wire is taken in the order of several hundred nanometers.

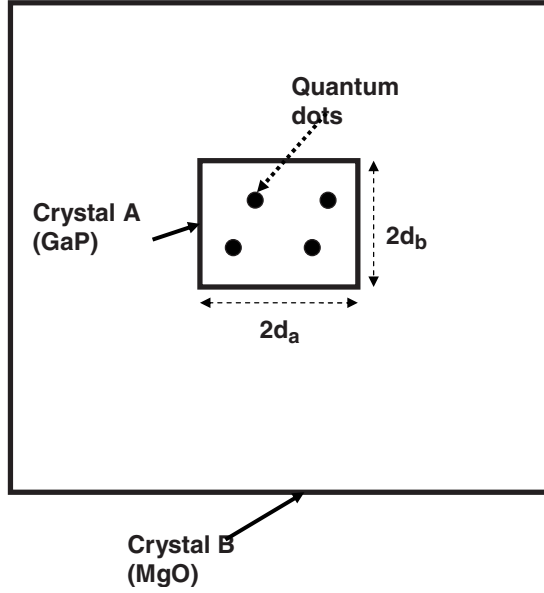


FIG. 1. A schematic diagram is shown for polaritonic nanowires (top view). The nanowire is made from GaP and MgO where GaP is embedded in MgO.

We have calculated the polaritonic bound states by using the transfer-matrix method. The density of states (DOS) for the polariton bound states has also been calculated. Numerical simulations are performed on a nanowire fabricated by embedding GaP into MgO. It has been found that the number of bound polaritons states depends on the size of the nanowire.

We have doped an ensemble of quantum dots in a polaritonic nanowire. There is a considerable interest in studying electronic and photonic properties of quantum dots doped in different materials.^{12,13} The quantum dots have been used as multilevel atoms to study the interference and coherence phenomena.^{12,13} The quantum dots are interacting with the polaritonic nanowire via the electron-bound polariton interaction. Three levels of the quantum dots are considered in the study of the absorption. A probe-laser field is applied to study the absorption process in the quantum dots. The absorption coefficient has been calculated by using the Schrodinger equation method.

Numerical simulations have been performed on the absorption coefficient and density of states. It has been found that the absorption spectrum splits into many absorption peaks due to the coupling between quantum dots and bound polariton states. The number of peaks in the absorption spectrum depends on the number of bound states in the nanowire. In other words, the absorption spectrum has several transparent and absorption states. The polaritonic nanowire can be switched from one transparent state to another. It can also be switched between a transparent minimum to an absorbing peak. This is a very interesting discovery which can be used to make polaritonic switches and transistors.

II. POLARITONIC NANOWIRES

We consider a polariton nanowire which is fabricated from two polaritonic materials, A and B. Material A is em-

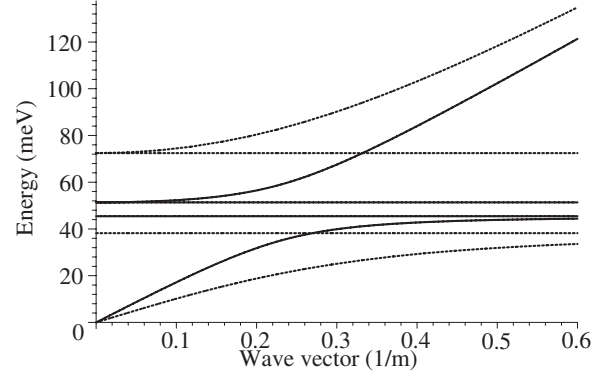


FIG. 2. The dispersion relation (i.e., energy vs wave vector) is plotted for GaP and MgO. The solid and dashed-dotted lines correspond to GaP and MgO, respectively. Note there is an energy gap in dispersion relations. The wave vector is multiplied by a constant 10^6 .

bedded into B. The dispersion relation for the polaritonic materials is written as⁴

$$k = \frac{\omega_k}{c} \sqrt{\epsilon_\infty \left(\frac{\omega_L^2 - \omega_k^2}{\omega_T^2 - \omega_k^2} \right)^{1/2}}, \quad (1)$$

where ω_L and ω_T are the longitudinal and transverse optical phonon frequencies of the polaritonic materials, respectively. The parameter ϵ_∞ is the dielectric constant when the photon frequency is infinite and c is the speed of light. The longitudinal frequency is related to the optical frequency by the Lyddane-Sachs-Teller relation as

$$\frac{\omega_L^2}{\omega_T^2} = \frac{\epsilon_0}{\epsilon_\infty}, \quad (2)$$

where ϵ_0 is the dielectric constant when the photon frequency is zero.

The dispersion relation has two solutions which are written as

$$\omega_k^2 = \left\{ \begin{array}{l} \frac{1}{2} \left[\omega_L^2 + \left(\frac{\hbar c k}{\sqrt{\epsilon_\infty}} \right)^2 \right] \\ \pm \frac{1}{2} \sqrt{\left[\omega_L^2 - \left(\frac{\hbar c k}{\sqrt{\epsilon_\infty}} \right)^2 \right]^2 + 4 \left(\frac{\hbar c k}{\sqrt{\epsilon_\infty}} \right)^2 (\omega_L^2 - \omega_T^2)} \end{array} \right\}. \quad (3)$$

Note from the above expressions that polaritonic materials have an energy gap in their dispersion relations. The energy gap lies between frequencies ω_L and ω_T .

Let us denote the upper band energy edge as $\epsilon_c = \hbar \omega_L$ and lower band energy edge as $\epsilon_v = \hbar \omega_T$. In the rest of the paper we will use the dispersion relation in the energy space rather than in the frequency space. Therefore, the dispersion relation of the crystals A and B can be written as

$$\varepsilon_{k_a}^{\pm} = \left\{ \begin{array}{l} \frac{1}{2} \left[(\epsilon_c^a)^2 + \frac{(\hbar c k_a)^2}{\epsilon_{\infty}} \right] \\ \pm \frac{1}{2} \sqrt{\left[(\epsilon_c^a)^2 - \frac{(\hbar c k_a)^2}{\epsilon_{\infty}} \right]^2 + \frac{(2\hbar c k_a)^2}{\epsilon_{\infty}} [(\epsilon_c^a)^2 - (\epsilon_v^a)^2]} \end{array} \right\}^{1/2}, \quad (4a)$$

$$\varepsilon_{k_b}^{\pm} = \left\{ \begin{array}{l} \frac{1}{2} \left[(\epsilon_c^b)^2 + \frac{(\hbar c k_b)^2}{\epsilon_{\infty}} \right] \\ \pm \frac{1}{2} \sqrt{\left[(\epsilon_c^b)^2 - \frac{(\hbar c k_b)^2}{\epsilon_{\infty}} \right]^2 + \frac{(2\hbar c k_b)^2}{\epsilon_{\infty}} [(\epsilon_c^b)^2 - (\epsilon_v^b)^2]} \end{array} \right\}^{1/2}, \quad (4b)$$

where (+) and (−) stand for the upper and lower polaritonic bands, respectively. Crystal A has an energy band gap between ϵ_c^a and ϵ_v^a and crystal B has an energy band gap between ϵ_c^b and ϵ_v^b .

In fabricating the polaritonic nanowire it is considered that the band gap of crystal A lies within the band gap of crystal B. This band-structure engineering will confine the polaritons in the doped crystal A. Therefore, crystal A acts as a polaritonic nanowire and the polaritons are confined within crystal A.

To achieve the above band-structure engineering we have chosen materials GaP and MgO as an example. The former is a semiconductor and the latter is an insulator. The upper and lower band edges for GaP are found experimentally as $\epsilon_c^a = 51.4$ meV and $\epsilon_v^a = 45.4$ meV, respectively.⁴ Similarly, the upper and lower band edges for MgO are found experimentally as $\epsilon_c^b = 72.5$ meV and $\epsilon_v^b = 38.2$ meV, respectively.⁴ The dielectric constant ϵ_{∞} is taken as 8.5 and 2.95 for GaP and MgO, respectively.⁴ Using these parameters the dispersion relations for both crystals GaP and MgO are plotted in Fig. 2. Solid lines correspond to GaP whereas dashed-dotted lines belong to MgO. Note that the upper and lower band edges of GaP lie within the band gap of MgO. See solid and dashed-dotted horizontal lines in Fig. 2. A schematic band structure of the polaritonic wire is plotted in Fig. 3.

Band-gap materials are generally quantified by the quantity called the gap to midgap ratio. This quantity plays an important role in photonic crystals when they are used to fabricate photonic devices. Photonic crystals are materials which also have band gaps in their photonic dispersion relation. It is defined as

$$r = \frac{\epsilon_c - \epsilon_v}{\frac{1}{2}(\epsilon_c + \epsilon_v)}. \quad (5)$$

The gap to midgap relation for GaP and MgO is found as 12.2% and 61.9%, respectively. Note that the ratio for MgO is larger than that of GaP.

The width and height of the wire A is taken as $2d_a$ and $2d_b$, respectively. The nanowire lies along the z direction and its length is taken as d_c . In Ref. 14 we have calculated the energy states of a nanowire made from photonic crystals. They are mainly manmade materials fabricated by arranging dielectric spheres in a periodic structure. We use the method

of Ref. 14 to calculate polariton energy states. Matching the boundary conditions for the wave functions at interfaces^{15–17} we obtained the following expressions for the energies of bound polaritons in the wire:

$$k_n \tan\left(k_n d_a - n \frac{\pi}{2}\right) - \sqrt{F_a^2(\epsilon) - F_b^2(\epsilon) - k_n^2} = 0, \quad (6a)$$

$$k_m \tan\left(k_m d_b - m \frac{\pi}{2}\right) - \sqrt{F_a^2(\epsilon) - F_b^2(\epsilon) - k_m^2} = 0, \quad (6b)$$

where k_n and k_m are the x and y components of the wave vector k_a . Parameters n and m are quantum numbers. Functions F_a and F_b are given as

$$F_a(\epsilon) = \frac{\epsilon \sqrt{\epsilon_{\infty}}}{\hbar c} \left[\frac{(\epsilon_c^a)^2 - \epsilon^2}{(\epsilon_v^a)^2 - \epsilon^2} \right]^{1/2}, \quad (7a)$$

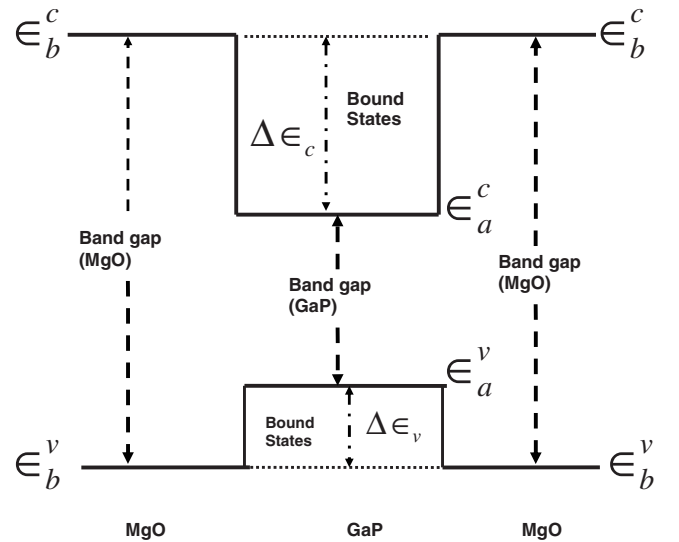


FIG. 3. A schematic band-structure diagram is plotted for a polaritonic nanowire. The lower band edges for GaP and MgO are denoted as ϵ_a^v and ϵ_b^v , respectively. Similarly the upper band edges for GaP and MgO are denoted as ϵ_a^c and ϵ_b^c , respectively. Note that the upper band of the crystal GaP has a lower value than that of MgO. The width and height of wire is taken as d_a and d_y .

$$F_b(\varepsilon) = \frac{\varepsilon \sqrt{\varepsilon_\infty} \left[\frac{(\varepsilon_c^b)^2 - \varepsilon^2}{(\varepsilon_v^b)^2 - \varepsilon^2} \right]^{1/2}}{\hbar c} \quad (7b)$$

The above expressions are obtained from Eqs. (4a) and (4b), respectively.

The wave vectors k_n and k_m are quantized. Therefore, the polariton energy within the wire is also quantized. Let the quantized energy for the upper and lower bands be denoted as $\varepsilon_{nmk_z}^+$ and $\varepsilon_{nmk_z}^-$, respectively. They are obtained from Eqs. (4a), (6a), and (6b) as

$$\varepsilon_{nmk_z}^+ = \left\{ \begin{array}{l} \frac{1}{2} \left[(\varepsilon_c^a)^2 + \frac{(\hbar c \sqrt{k_n^2 + k_m^2 + k_z^2})^2}{\varepsilon_\infty} \right] \\ + \frac{1}{2} \sqrt{\left[(\varepsilon_c^a)^2 - \frac{(\hbar c \sqrt{k_n^2 + k_m^2 + k_z^2})^2}{\varepsilon_\infty} \right]^2 + \frac{(2\hbar c \sqrt{k_n^2 + k_m^2 + k_z^2})^2}{\varepsilon_\infty} [(\varepsilon_c^a)^2 - (\varepsilon_v^a)^2]} \end{array} \right\}^{1/2}, \quad (8a)$$

$$\varepsilon_{nmk_z}^- = \left\{ \begin{array}{l} \frac{1}{2} \left[(\varepsilon_c^a)^2 + \frac{(\hbar c \sqrt{k_n^2 + k_m^2 + k_z^2})^2}{\varepsilon_\infty} \right] \\ - \frac{1}{2} \sqrt{\left[(\varepsilon_c^a)^2 - \frac{(\hbar c \sqrt{k_n^2 + k_m^2 + k_z^2})^2}{\varepsilon_\infty} \right]^2 + \frac{(2\hbar c \sqrt{k_n^2 + k_m^2 + k_z^2})^2}{\varepsilon_\infty} [(\varepsilon_c^a)^2 - (\varepsilon_v^a)^2]} \end{array} \right\}^{1/2}, \quad (8b)$$

where the component k_z is the z component of k_a and is not quantized. It is responsible for the polariton propagation along the wire.

The expression of k_z is obtained from Eq. (1) as ε_{nmk_z} ,

$$k_z = \sqrt{F_a^2(\varepsilon_{nmk_z}^\pm) - (k_{nm}^2)}, \quad (9a)$$

where

$$k_{nm} = \sqrt{(k_n)^2 + (k_m)^2},$$

$$F_a(\varepsilon_{nmk_z}^\pm) = \frac{(\varepsilon_{nmk_z}^\pm)^2 \sqrt{\varepsilon_\infty} \left[\frac{(\varepsilon_c^a)^2 - (\varepsilon_{nmk_z}^\pm)^2}{(\varepsilon_v^a)^2 - (\varepsilon_{nmk_z}^\pm)^2} \right]^{1/2}}{\hbar c} \quad (9b)$$

When $k_z=0$, the quantized energy of the wire can be calculated from the following equation:

$$k_{nm} = F_a(\varepsilon_{nm}). \quad (10)$$

The expression of k_z further reduces to

$$k_z = \sqrt{F_a^2(\varepsilon_{nmk_z}^\pm) - F_a^2(\varepsilon_{nm})}. \quad (11)$$

We found that the polariton energies are quantized within the wire. That is why we call these structures nanopolaritonic wires.

III. ABSORPTION COEFFICIENT

It is considered that the nanowire is doped lightly with quantum dots. Therefore, the interaction between the quantum dots can be neglected. Three energy levels of the quantum dots are considered in the calculation of the absorption coefficient. The levels are denoted by $|a\rangle$, $|b\rangle$, and $|c\rangle$ where $|a\rangle$ is the ground state. A schematic diagram of a quantum dot is presented in Fig. 4.

We consider that the quantum dots are interacting with the polaritonic nanowire which is acting as a reservoir. We call this the quantum-dot-bound polariton (QDBP) interaction. The origin of this interaction is as follows. We consider that the transition energy for the transition $b \leftrightarrow c$ lies near one of the polariton localized (bound) states of the nanowire. This transition induces an electronic dipole in the quantum dots which interact with the polariton bound states. This interaction is nothing but the electron-bound polariton interaction.

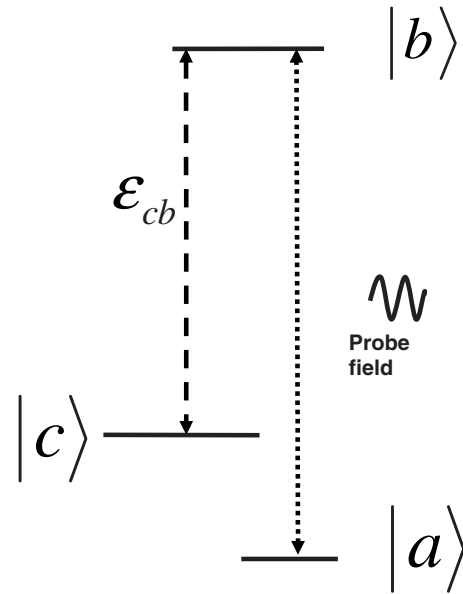


FIG. 4. A schematic diagram of a three-level quantum dot. Three levels are denoted by $|a\rangle$, $|b\rangle$, and $|c\rangle$ where $|a\rangle$ is the ground state. A probe field is applied to monitor the absorption transition $a \leftrightarrow b$. The transition $b \leftrightarrow c$ couples to the polaritonic bound states of the wire.

We have calculated this interaction for three-dimensional polaritonic materials in Ref. 1. Following the method of these references we have obtained the Hamiltonian for the interacting quantum dots with bound polaritons as

$$H_{pol} = H_0 + H_{QDBP}, \quad (12a)$$

where

$$H_0 = \sum_{nm} \sum_{k_z} \varepsilon_{nmk_z}^+ p_+^\dagger(nmk_z) p_+(nmk_z) + \sum_{nm} \sum_{k_z} \varepsilon_{nmk_z}^- p_-^\dagger(nmk_z) p_-(nmk_z), \quad (12b)$$

$$H_{QDBP} = - \sum_{nm} \sum_{k_z} V_+(\varepsilon_{nmk_z}) p_+(nmk_z) \sigma_{bc}^+ e^{i(\varepsilon_{bc} - \varepsilon_{nmk_z}^+)t/\hbar} - \sum_{nm} \sum_{k_z} V_-(\varepsilon_{nmk_z}) p_-(nmk_z) \sigma_{bc}^+ e^{i(\varepsilon_{bc} - \varepsilon_{nmk_z}^-)t/\hbar} + \text{H.c.}, \quad (12c)$$

where (+) and (-) stand for the upper and lower polaritonic bands, respectively. The operator σ_{bc}^+ is called the raising operator and is defined as $\sigma_{bc}^+ = |c\rangle\langle b|$. The resonance energy ε_{bc} is the energy difference between $|b\rangle$ and $|c\rangle$ and H.c. stands for the Hermitian conjugate. Here p_\pm^\dagger and p_\pm are called the polariton annihilation and creation operators, respectively. The first and second terms in the Hamiltonian correspond to the bound polaritons and the QDBP interaction, respectively.

Functions $V_\pm(\varepsilon_{nmk_z}^\pm)$ are the polariton coupling parameters and are written as

$$V_+ = \sqrt{\frac{ck\mu_{bc}^2}{6\hbar Ad_c \varepsilon_0}} \left[\frac{(\varepsilon_{nmk_z}^+)^2 - \varepsilon_c^2}{(\varepsilon_{nmk_z}^+)^2 - (\varepsilon_{nmk_z}^-)^2} \right]^{1/2},$$

$$V_- = \sqrt{\frac{ck\mu_{bc}^2}{6\hbar Ad_c \varepsilon_0}} \left[\frac{\varepsilon_c^2 - (\varepsilon_{nmk_z}^-)^2}{(\varepsilon_{nmk_z}^+)^2 - (\varepsilon_{nmk_z}^-)^2} \right]^{1/2}, \quad (13)$$

where k is the wave vector with its expression given in Eq. (1). Here A is the cross-sectional area of the nanowire and μ_{bc} is the dipole moment for the transition $b \leftrightarrow c$. Note that polariton coupling depends on the cross-sectional area A .

$$s(\varepsilon_{nmk_z}^\pm) = \frac{[\varepsilon_v^2 - (\varepsilon_{nmk_z}^\pm)^2]^{1/2} [\varepsilon_c^2 - 2(\varepsilon_{nmk_z}^\pm)^2] + (\varepsilon_{nmk_z}^\pm)^2 [\varepsilon_c^2 - (\varepsilon_{nmk_z}^\pm)^2]}{(\hbar c / \sqrt{\varepsilon_\infty}) [\varepsilon_c^2 - (\varepsilon_{nmk_z}^\pm)^2]^{1/2} [\varepsilon_v^2 - (\varepsilon_{nmk_z}^\pm)^2]}. \quad (19)$$

Here Φ is called the theta function and has the following property:

$$\Phi(\varepsilon_{nmk_z}^\pm - \varepsilon_{nm}^\pm) = 1 \quad \text{for} \quad (\varepsilon_{nmk_z}^\pm - \varepsilon_{nm}^\pm) > 1,$$

$$\Phi(\varepsilon_{nmk_z}^\pm - \varepsilon_{nm}^\pm) = 0 \quad \text{for} \quad (\varepsilon_{nmk_z}^\pm - \varepsilon_{nm}^\pm) < 1. \quad (20)$$

The expression of the DOS can be simplified when the polariton energy lies near one of the bound energies ε_{nm}^\pm . We

A probe-laser field is applied to monitor the absorption process due to the transition $a \leftrightarrow b$ in the quantum dots. The energy of the probe field is assigned the letter ε_p . We use the time-dependent Schrödinger equation to calculate the absorption coefficient. Following the method of Ref. 14 and using Eq. (12) we have found the following expression of the susceptibility in the linear-response theory:

$$\chi = -\chi_0 \left(\frac{\gamma_b}{\delta_{ab} + i\gamma_b + \Xi_{bc}^+ + \Xi_{bc}^-} \right), \quad (14)$$

where

$$\chi_0 = \left(\frac{N\mu_{ab}^2}{\hbar \varepsilon_0 \gamma_b} \right).$$

Here γ_b and γ_c are the linewidths of levels $|b\rangle$ and $|c\rangle$ in the absence of the QDBP interaction, respectively. μ_{ab} is the dipole moment for the transition $a \leftrightarrow b$. The parameter δ_{ab} is called the probe detuning and is defined as

$$\delta_{ab} = \varepsilon_{ab} - \varepsilon_p.$$

Functions Ξ_{bc}^\pm are called self-energies and are obtained as

$$\Xi_{bc}^\pm = \sum_{nm} \sum_{k_z} \frac{i|V_\pm(\varepsilon_{nmk_z}^\pm)|^2}{\gamma_c - i(\delta_{ab} - \varepsilon_{nmk_z}^\pm + \varepsilon_{ca})}. \quad (15)$$

Using the concept of the DOS, the summation over k_z can be replaced by the integration over energy $\varepsilon_{nmk_z}^\pm$ as

$$\sum_{n,m} \sum_{k_z} = \sum_{n,m} \int \rho_{nm}^\pm(\varepsilon_{nmk_z}^\pm) d\varepsilon_{nmk_z}^\pm, \quad (16)$$

where ρ_{nm}^\pm is the DOS and is defined as

$$\rho_{nm}^\pm = \frac{d_c}{\pi} \frac{dk_z}{d\varepsilon_{nmk_z}^\pm}. \quad (17)$$

With the help of Eq. (11) the DOS is evaluated as

$$\rho_{nm}^\pm = \frac{d_c}{\pi} \frac{F_a(\varepsilon_{nmk_z}^\pm) s(\varepsilon_{nmk_z}^\pm)}{\sqrt{F_a^2(\varepsilon_{nmk_z}^\pm) - F_a^2(\varepsilon_{nm}^\pm)}} \Theta(\varepsilon_{nmk_z}^\pm - \varepsilon_{nm}^\pm), \quad (18)$$

where

expand the function $F^2(\varepsilon_{nmk_z}^\pm - \varepsilon_{nm}^\pm)$ in Eq. (18) near $\varepsilon_{nmk_z}^\pm = \varepsilon_{nm}^\pm$ by using the Taylor series

$$F_a^2(\varepsilon) = F_a^2(\varepsilon)|_{\varepsilon=\varepsilon_{nm}^\pm} + \left. \frac{dF_a^2(\varepsilon)}{d\varepsilon} \right|_{\varepsilon=\varepsilon_{nm}^\pm} (\varepsilon - \varepsilon_{nm}^\pm). \quad (21)$$

The first term in the above equation has the following form

$$F_a^2(\varepsilon)|_{\varepsilon-\varepsilon_{nm}^\pm} = F_a^2(\varepsilon_{nm}^\pm). \quad (22)$$

The second term in Eq. (21) can be evaluated using Eq. (9b) after which we obtain

$$F_a^2(\varepsilon_{nmk_z}^\pm) = F_a^2(\varepsilon_{nm}^\pm) + 2F_a(\varepsilon_{nm}^\pm)s(\varepsilon_{nm}^\pm)(\varepsilon_{nmk_z}^\pm - \varepsilon_{nm}^\pm). \quad (23)$$

Finally the DOS is calculated as

$$\rho_{mp}^\pm = \left[\frac{d_c F_a(\varepsilon_{nmk_z}^\pm)s(\varepsilon_{nmk_z}^\pm)}{\pi \sqrt{2F_a(\varepsilon_{nm}^\pm)s(\varepsilon_{nm}^\pm)}} \right] \frac{\Theta(\varepsilon_{nmk_z}^\pm - \varepsilon_{nm}^\pm)}{\sqrt{(\varepsilon_{nmk_z}^\pm - \varepsilon_{nm}^\pm)}}. \quad (24)$$

The above expression can be further simplified if we put $\varepsilon_{nmk_z}^\pm = \varepsilon_{nmk_z}^\pm$ in the numerator. The DOS reduces to

$$\rho_{mp}^\pm = \frac{\beta_{nm}^\pm}{\sqrt{(\varepsilon_{nmk_z}^\pm - \varepsilon_{nm}^\pm)}} \Theta(\varepsilon_{nmk_z}^\pm - \varepsilon_{nm}^\pm), \quad (25)$$

where

$$\beta_{nm}^\pm = \frac{d_c}{\sqrt{2\pi}} \sqrt{Q_{nm}^\pm},$$

$$Q_{nm}^\pm = \sqrt{F_a(\varepsilon_{nm}^\pm)s(\varepsilon_{nm}^\pm)}. \quad (26)$$

It is important to note that the DOS of the nanowires has a singularity at $\varepsilon_{nmk_z}^\pm = \varepsilon_{nm}^\pm$.

In terms of the DOS, the self-energy reduces to

$$\Xi_{bc}^\pm = \sum_{m,p} \int_{-\infty}^{\infty} \frac{i|V_\pm(\varepsilon_{nmk_z}^\pm)|^2 \rho_{mp}^\pm}{\gamma_c - i(\delta_{ab} - \varepsilon_{nmk_z}^\pm + \varepsilon_{ca})}. \quad (27)$$

Putting the expression of the DOS from Eq. (25) into the above expression we find

$$\Xi_{bc}^\pm = \sum_{nm} \int_{\varepsilon_{nm}^\pm}^{\infty} \frac{\gamma_{nm}^\pm d\varepsilon_{mpk_z}^\pm}{[\gamma_c - i(\delta_{ab} - \varepsilon_{nmk_z}^\pm + \varepsilon_{bc})] \sqrt{(\varepsilon_{nmk_z}^\pm - \varepsilon_{nm}^\pm)}}, \quad (28)$$

where

$$\gamma_{nm}^\pm = \beta_{nm}^\pm |V_\pm(\varepsilon_{nm}^\pm)|^2. \quad (29)$$

The expression of the self-energy can be further simplified to

$$\Xi_{bc}^\pm = \sum_{nm} \frac{-\gamma_{nm}^\pm}{\sqrt{i\gamma_c + (\delta_{ab} - \delta_{nm}^\pm)}}, \quad (30)$$

where δ_{nm}^\pm is called the bound polariton parameter. This parameter measures the difference between the resonance energy ε_{bc} and the polariton bound energies ε_{nm}^\pm and is defined as

$$\delta_{nm}^\pm = \varepsilon_{nm}^\pm - \varepsilon_{bc}. \quad (31)$$

The parameter γ_{nm}^\pm is called the QDBP parameter and is found as

$$\gamma_{nm}^+ = \frac{3\pi\hbar^3 c^3 \gamma_0}{\sqrt{2\varepsilon_{bc}^2}} \left[\frac{\varepsilon_c^2 - (\varepsilon_{nmk_z}^+)^2}{(\varepsilon_{nmk_z}^+)^2 - (\varepsilon_{nmk_z}^-)^2} \right] \left[\frac{\sqrt{Q(\varepsilon_{nm}^+)}}{d_a d_b} \right],$$

$$\gamma_{nm}^- = \frac{3\pi\hbar^3 c^3 \gamma_0}{\sqrt{2\varepsilon_{bc}^2}} \left[\frac{(\varepsilon_{nmk_z}^-)^2 - \varepsilon_c^2}{(\varepsilon_{nmk_z}^+)^2 - (\varepsilon_{nmk_z}^-)^2} \right] \left[\frac{\sqrt{Q(\varepsilon_{nm}^-)}}{d_a d_b} \right], \quad (32)$$

where γ_0 is the linewidth of an atom in a vacuum. The QDBP parameter controls the coupling between a quantum dot and bound polaritons.

Note that the polariton absorption coefficient depends on the self-energy which in turn depends on the energy difference between the transition energy ε_{bc} and a bound polariton energy ε_{nm} (i.e., δ_{nm}). It also relies on the polariton coupling constant γ_{nm}^\pm .

IV. NUMERICAL SIMULATIONS AND DISCUSSIONS

In this section we consider that the polaritonic nanowire consists of GaP and MgO. These two materials are chosen because the band gap of GaP lies within the band gap of MgO. The crystal parameters for crystals of GaP are taken as $\varepsilon_c^a = 51.4$ meV and $\varepsilon_v^a = 45.4$ meV.⁴ Similarly the crystal parameters for crystals of MgO are taken as $\varepsilon_c^a = 72.5$ meV and $\varepsilon_v^a = 38.2$ meV.

Note that the upper band of GaP is higher than that of MgO. Let us define the well depth of the wire as $\Delta\varepsilon_c = \varepsilon_c^b - \varepsilon_c^a$. The well depth of the present wire is $\Delta\varepsilon_c = 21.1$ meV. Therefore, polaritons whose energies lie within the well depth are bound according to our theory.

Note also that the lower band edge of GaP is higher than that of MgO. We define a barrier height of the wire as $\Delta\varepsilon_v = \varepsilon_v^a - \varepsilon_v^b$. Then, the barrier height of the present wire is found as $\Delta\varepsilon_v = 7.2$ meV. Therefore, polaritons whose energies lie within the barrier height are bound within the barrier. See Fig. 3.

We have performed the numerical simulation for bound polariton energies using the equations derived in Sec. II. The size of the nanowires are taken as $d_a = d_b = 1500$ nm. Let us first calculate bound polariton states lying within the well depth formed by the upper bands of GaP and MgO. In Fig. 5 we have plotted the function appearing to the left-hand side of Eq. (6). Let us call it the BP function. This function gives the number of bound states within the wire. Note in the figure that the BP function with $n=0$ cuts the x axis whereas the BP function with $n=1$ does not cut the x axis. This means that the wire has one bound state with $n=m=0$ and is located at energy $\varepsilon_{00} = 66.3$ meV. Here ε_{00} stands for the ground state.

Let us increase the size of the wire as $d_a = d_b = 2500$ nm. Now the results are plotted in Fig. 6. In this case both BP functions with $n=0$ and 1 cut the horizontal axis. This gives two bound states within the wire with energies $\varepsilon_{00} = 57.5$ meV and $\varepsilon_{10} = 71.3$ meV. The bound-state energy ε_{10} corresponds to the state with quantum numbers $n=1$ and $m=0$. However, it is found that the bound state ε_{10} is degenerate with the bound state ε_{01} which corresponds to quantum

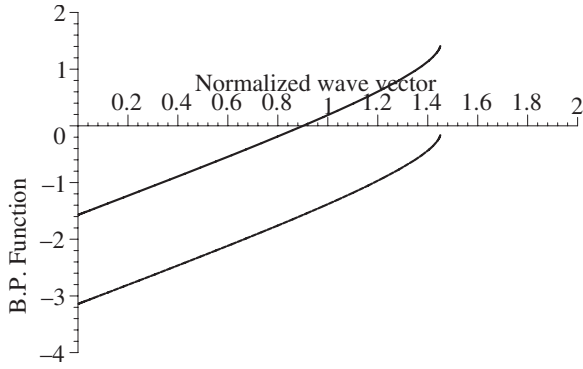


FIG. 5. The BP function is plotted as function of the normalized wave vector. The size of the nanowire is taken as $d_a=d_b=1500$ nm. Note that solid line with $n=0$ becomes zero for a given value of the quantized wave vector. However the dotted curve for $n=1$ does not have a zero value.

numbers $n=0$ and $m=1$. They are degenerate because we have considered a symmetric nanowire. Note that both bound states lie within the energy depth. From Figs. 6 and 7 we conclude that the number of bound states depends on the cross-sectional area of the wire.

Let us measure the ground-state energy with respect to the upper band edge ϵ_a^c . The ground-state energy ϵ_{00} becomes $\Delta\epsilon_{00}=\epsilon_{00}-\epsilon_a^c=14.9$ meV. It is interesting to note that there are no polaritonic energy states between the ground state and the upper band edge. This means that the upper band edge of the nanowire has been increased from $\epsilon_a^c=51.4$ meV to $\epsilon_a^c+\Delta\epsilon_{00}=66.3$ meV. The increase in the band edge depends on the size of the wire.

We have also calculated the bound polaritonic states within the barrier of the nanowire. When the size of the wire is taken as $d_a=d_b=2500$ nm, we found several bound states. They are located at $\epsilon_{00}=39.3$ meV, $\epsilon_{10}=43.5$ meV, and $\epsilon_{11}=44.4$ meV. The higher excited states are almost degenerate with excited state ϵ_{11} . The reason it has several bound states is because the height of the barrier is very small. Note that there are no polaritonic energy states between the ground state and the lower band edge.

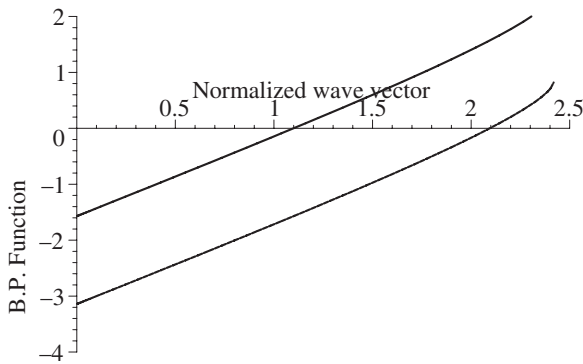


FIG. 6. The BP function is plotted as a function of the normalized wave vector. The size of the nanowire is taken as $d_a=d_b=2500$ nm. Note that solid line with $n=0$ becomes zero for a given value of the quantized wave vector. The dotted curve for $n=1$ also has a zero root. The system has two bound states.

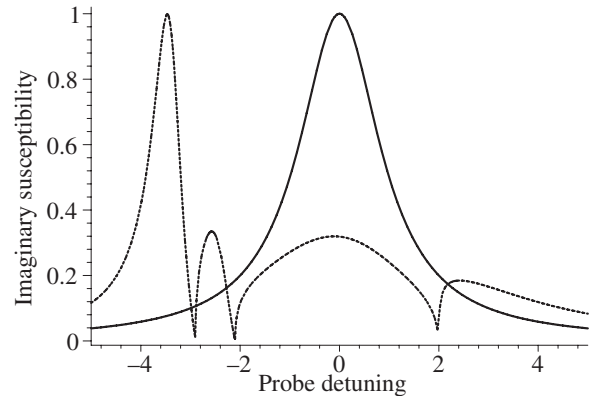


FIG. 7. The plot of the absorption coefficient $\text{Im}(\chi/\chi_0)$ versus the detuning parameter δ_{ab} . The solid and dotted curves are plotted in the absence and presence of the QDBP interaction, respectively.

Next, we calculate the absorption coefficient in the polaritonic wire due to the transition $|a\rangle \leftrightarrow |b\rangle$. The effect of the QDBP interaction is included in the absorption coefficient. The energies are measured in terms of γ_b which is assigned a value of 1.0 meV.¹⁴ The linewidth γ_c is taken as $0.001\gamma_b$.¹⁴ We considered that the resonance energy lies near the bound states i.e., $\epsilon_{bc}=41.2$ meV.

The absorption coefficient is calculated for the polaritonic barrier which is formed due to the lower band of GaP and MgO. The results are plotted in Fig. 7 as a function of the probe detuning parameter. The solid and dotted curves denote the cases in the absence and the presence of the QDBP interaction, respectively. Note that the absorption spectrum displays the usual symmetric Lorentzian line shape in the absence of the interaction. The maximum of the absorption peak is located at zero detuning.

However, in the presence of the interaction the peak splits into several peaks. This means that the interaction is strong and is able to split the absorption spectrum. The interaction is strong since the DOS of the polaritonic nanowire is very large at the bound states. The DOS has been plotted in Fig. 8 as a function of energy. One can see that the DOS is very large near the bound photon states ϵ_{00} , ϵ_{10} , and ϵ_{11} .

The several transparent minima in the absorption spectrum are due to the scattering of the quantum dots with sev-

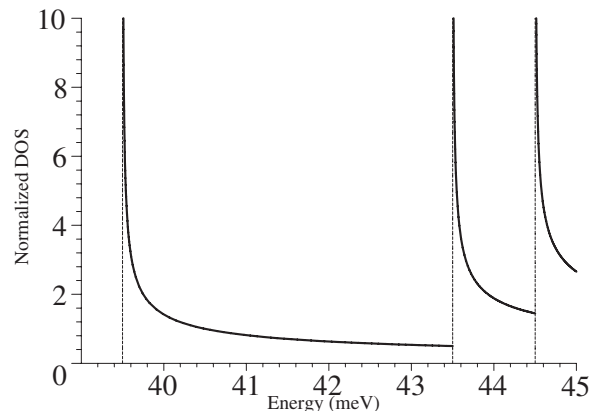


FIG. 8. The density of states is plotted as a function energy.

eral bound states in the wire. The splitting of the absorption peaks is explained by using the physics of dressed states.¹⁸ In summary we have found several transparent and absorption states in the nanowire. The polaritonic nanowire can be switched among transparent and absorption states.

Finally we comment on the configuration of nanowires. In the present paper we have used a square nanowire for the calculation of bound photon energies. The bound photon energies have also been calculated for a circular nanowire. It is found the number of bound states and their energies depend on the radius of the wire. However, the finding of the paper remains the same. The bound photon wave functions for the square nanowire consist of sine and cosine functions. On the other hand, the wave functions for the circular wire are made from Bessel functions. Photon bound states are degenerate in the square and circular nanowires because of their symmetrical configurations. However, in elliptical nanowires the degeneracy of the bound photon states is removed because of their asymmetric configurations.

V. CONCLUSIONS

We have studied the optoelectronics of a polaritonic nanowire doped with an ensemble of three-level quantum dots.

The wire is made from two polaritonic crystals, A and B, where crystal A is embedded in crystal B. Crystal A has a smaller band gap than that of crystal B. Because of this band-structure engineering the polaritons are confined in crystal A. The bound states of the confined polaritons are calculated. It is found that the number of bound states in the wire depends on its size, well depth, and the barrier height. The absorption coefficient of the system has also been calculated using the time-dependent Schrödinger equation method. Numerical simulations show when the resonance energy for one of the transitions lies near one of the bound states the spectrum has several transparent states. The nanowire can be switched among the transparent or absorbing states by tuning the resonance state of the quantum dot. The present findings can be used to make devices such as polaritonic switches and transistors.

ACKNOWLEDGMENTS

This work was supported by the Natural Sciences and Engineering Research Council of Canada (NSERC).

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